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Time Delay Cosmography - Peering Through a Cosmic Telescope

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ABSTRACT

Albert Einstein shook the very foundations of physics when he proposed his general theory of relativity. The idea that mass warps spacetime was promptly verified: the paths of photons are curved around massive objects. This effect is not simply a quirk of the cosmos, but a powerful tool. By chance alignment, a galaxy cluster of immense mass can act as a gravitational lens, focusing the light of extremely distant galaxies into multiple magnified images. These images reveal details that even our most advanced instruments would be incapable of resolving without the gravitational lens. Sjur Refsdal was the first to note that the photons that form separate images of the same source take paths of different lengths through the lens, and there is therefore a time delay between the images. In a 1964 paper he showed succinctly that if this time delay could be measured, the Hubble Constant could be constrained with systematics that are independent from other methods in cosmology. Only in the last decade has Refsdal's technique of "time delay cosmography" become possible for the first time. Can the time delays of gravitationally lensed supernovae crack the crisis in cosmology and heal the Hubble tension? Gaze with me through the cosmic telescopes as I explore the theory, observations, and results of time delay cosmography.

1. INTRODUCTION

Supernovae (SNe) are among the most luminous astrophysical transients. These explosions mark the deaths of stars, and can briefly outshine their entire host galaxies. Of particular interest to astronomers are the oldest and most distant SNe. Observations of these high redshift SNe are crucial to our understanding of the early universe; they can provide insight into the environments of galaxies at those primitive epochs and allow us to estimate the expansion rate of the universe. Alas, the universe is immense in scale, and despite their intrinsically high luminosity, SNe are rarely seen if they occur at a higher redshift than $z \sim 1$. These SNe are too far away for modern surveys to detect. Further complicating their detection, the peak of their spectral energy distribution has been shifted from the optical to the infrared, where Earth's atmosphere more severely inhibits observation. However, through a peculiar prediction of general relativity, there is a way to see these ancient explosions: we must use the most massive galaxy clusters in the universe as cosmic telescopes to peer back further in space and time.

At its most basic level, general relativity says that mass warps spacetime. Thus, if a sufficiently massive cluster of galaxies sits in front of an extremely distant galaxy from our point of view on Earth, the light from that distant galaxy can follow a warped path around the edges of the galaxy cluster, magnifying that distant source. This convenient effect was predicted by Einstein in 1936 (Einstein 1936), but was first observed in 1979 (Walsh et al. 1979). Through repeated observations of a distant galaxy magnified by a foreground gravitational lens, astronomers can search for changes in the images to identify SNe. These SNe are occurring in such distant reaches of the universe that without the lens, they likely would have gone undetected. By analyzing the properties of these lensed SNe, information about galaxies in the early universe can be retrieved, and the Hubble Constant (H_0) can be measured.

To obtain this valuable information, unique challenges must be overcome. When taking astronomical measurements, the telescope must first be calibrated and characterized. Usually, a telescope is built by humans who painstakingly document the properties of the telescope they are constructing so that the image it produces can be thoroughly understood and the effects of the instrument itself can be separated from the true properties of the astronomical source. When the source is magnified not by a carefully crafted telescope, but instead by an inhomogeneous distribution of massive galaxies, great care must be taken to understand the information conveyed by the observation. The galaxy cluster acting as a gravitational lens must be modeled with precision to predict how light from a source behind the lens will interact with it. The goal of this term paper is to detail this modeling, and describe how it has been used to extract cosmologically pertinent information from specific gravitationally lensed transient events.

The rest of this paper will be laid out as follows: In section 2 I will discuss the physics of gravitational lenses, the various types of images they can produce, and the modeling techniques used to predict these effects. Section 3 will pertain to the relevance of lensed SNe to cosmology and what these lensed SNe can tell us about the universe through the technique of time delay cosmography. The 4th section will provide an in-depth discussion of four observed lensed SNe and the cosmological conclusions drawn from them. In section 5 I will examine the future of lensed transient studies in the age of deep all-sky surveys. Section 6 will conclude the paper.

2. MODELING THE PHYSICS OF GRAVITATIONAL LENSES

2.1. *Lens Geometry and Image Formation*

Albert Einstein's general theory of relativity predicts that mass warps the space and time around it. Photons follow the shortest paths through this curved spacetime, called null geodesics. This hypothesis was readily testable during a solar eclipse in 1919, when a star positioned directly behind the sun was observed due to the deflection of its light by the sun's gravitational field (Dyson et al. 1920). Gravitational lenses work by the same principle, but with masses and distances that are orders of magnitude higher. On cosmological scales, the object acting as a lens is generally a massive cluster of galaxies, and the lensed object is a high redshift galaxy situated behind the massive cluster. The geometry of this scenario is shown in Figure 1(a). Light from the distant source can take various paths around the lens and be deflected to Earth, resulting in a warped and magnified image of the source. In many cases, the geometry and gravitational potentials involved result in multiple images of the distant source. For a lens with a known gravitational potential Φ, the deflection angle $\hat{\alpha}$ can be calculated:

$$\hat{\alpha} = 2 \int ds \nabla_{\perp} (\frac{\Phi}{c^2}) \qquad \text{(16.1) Bovy (2025)}$$

When the angles involved are small, which is a good assumption in this case, β can be defined as

$$\beta = \theta - \frac{D_{LS}}{D_S} \hat{\alpha} \qquad \text{(16.13) Bovy (2025)}$$

Another useful quantity to define is the lensing potential $\psi(\theta)$, where

$$\psi(\theta) = \frac{D_{LS}}{D_S D_L} \frac{2}{c^2} \int dz \Phi(D_L, \theta, z) \qquad \text{(16.26) Bovy (2025)}$$

This is a useful quantity as it allows us to define the Fermat potential τ.

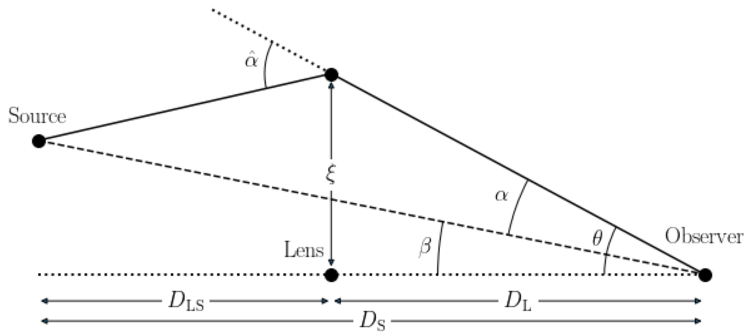
$$\tau(\theta, \beta) = \frac{1}{2} |\theta - \beta|^2 - \psi(\theta) \qquad \text{(16.43) Bovy (2025)}$$

The extrema of τ allow for the determination of the duplicity of the images. That is, the number of minima and maxima of τ is the number of images of the source that the lens will form (Bovy 2025). Duplicated images that are closer together are more highly magnified, and as these images are brought together this magnification diverges to infinity, resulting in a smear of the the highly magnified source along a contour called a critical curve (Bovy 2025). For a source perfectly aligned with an axially symmetric lens, the image of the source fully wraps around the lens, creating an Einstein Ring. An example of this phenomenon is seen in Figure 1(b).

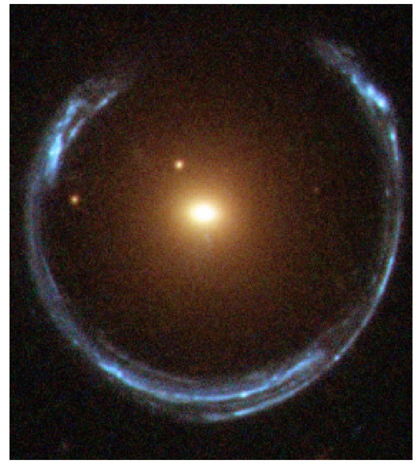
2.2. *The Modeling of Specific Gravitational Lenses*

A frequently used model of gravitational lenses is the Light-Traces-Mass (LTM) model. As the name suggests, this model assumes that the mass of a galaxy in the cluster directly corresponds to the luminosity of that galaxy (Frye et al. 2024). Furthermore, it is assumed that dark matter also follows this distribution. A power law is used to describe the mass surface density of each member galaxy of the cluster, with the weight of each power law set by the galaxy's measured luminosity. Gaussian

Figure 1.



(a) Shown here is the geometry of a gravitational lens. θ is the angular displacement of the lensed source from the center of the lens and is a measurable quantity. $\hat{\alpha}$ is the angle by which light from the source has been deflected from the direction in which it was emitted. If there was no lens, the light would have continued on its original path and the source would instead appear at an angle β. D_s is the distance between Earth and the source, while D_L is the distance between Earth and the lens, and D_{LS} is the distance from the lens to the source. Adapted from Bovy (2025).



(b) An Einstein ring as seen by the Hubble Space Telescope. A foreground elliptical galaxy magnifies and warps a background galaxy into a near perfect ring. Nemiroff & Bonnell (2011)

smoothing is then applied to the mass surface density to account for the diffuse nature of dark matter halos (Frye et al. 2024; Pascale et al. 2025). This model does not differentiate between non-luminous baryonic matter and dark matter. Rather, all matter that is not emitting light is treated as dark matter (Frye et al. 2024). Aside from galaxy luminosities and their positions in the sky, another input for this model are the redshifts of each galaxy in the cluster. These are derived from the doppler shift of known stellar absorption lines in each individual cluster member. For galaxies without spectroscopic redshifts, the redshift is left as a free parameter. The final inputs for the model relate to the source whose image is magnified and distorted by the lens. The redshift of the source, the location of the images and their magnifications can also help constrain the properties of the lens (Frye et al. 2024).

Now you may be asking yourself, if the images of the source are an input for the model, what exactly is the model predicting? By describing the nature of the lens in detail, a very useful property can be derived: the time delay between images. Especially around inhomogeneous or asymmetric lenses, the null geodesic that a photon from from the source travels to form one image can be a different length than the null geodesic that a different photon takes to form a separate image. The difference in the length of these paths results in a time delay between two different images, where a change in the source such as a supernova in the source galaxy will be first visible in the image of the source with the shortest null geodesic. That same supernova will appear at a later time in an image of the source that follows a longer null geodesic (Refsdal

1964; Frye et al. 2024; Pascale et al. 2025). The usefulness of the measurement of the time delay between these images is discussed in the following section.

3. TIME DELAY COSMOGRAPHY

In 1964, Norwegian astrophysicist Sjur Refsdal realized that the time delay between images formed by a gravitational lens could be expressed mathematically using only known or measurable quantities and H_0 (Refsdal 1964). Specifically, the time delay between two images, i and j, can be expressed as

$$\Delta t_{i,j} = \frac{1+z_L}{c}\frac{D_L D_S}{D_{LS}}\Delta\tau(\theta_i,\theta_j,\beta) \qquad (2)$$ Pascale et al. (2025)

Where $\Delta\tau(\theta_i,\theta_j,\beta)$ is the difference between the Fermat potentials for each image. The key to retrieving H_0 from the measurement of $\Delta t_{i,j}$ is that D_L, D_S, and D_{LS} are angular diameter distances, and are therefore dependent on the cosmological model used (Refsdal 1964; Pascale et al. 2025). It should be noted that the Fermat Potential depends on the lens model used, which also relies on a cosmological model. A "fiducial" value of H_0 (H_0^{fid}) is plugged into the lens model, yielding a "fiducial" prediction for the time delay ($\Delta t_{i,j}^{fid}$). After the true value for the time delay ($\Delta t_{i,j}$) is measured, the implied H_0 is related to the "fiducial" values like so:

$$H_0 = \frac{\Delta t_{i,j}^{fid}}{\Delta t_{i,j}} H_0^{fid} \qquad (3)$$ Pascale et al. (2025)

This technique provides a method of deducing H_0 that is wholly separate from those derived from SN Ia cosmology and CMB cosmology (Suyu et al. 2024). Because of this independence from other methods, time delay cosmography is an extremely valuable tool for measuring the expansion of the universe.

It should be noted that there is a prominent degeneracy in this technique: if a uniform sheet of matter is overlaid on the field of the cluster, the predicted positions, shapes, and magnifications of the image are unchanged (Frye et al. 2024). However, the time delays (and H_0 measurements derived therefrom) are affected by this "mass-sheet degeneracy". Breaking this degeneracy requires precise knowledge of the absolute magnitude of the lensed source (Frye et al. 2024; Pascale et al. 2025)

4. OBSERVED GRAVITATIONALLY LENSED SUPERNOVAE

Sjur Refsdal derived the foundations of time delay cosmography well before the detection of any gravitational lenses, let alone the detection of a gravitationally lensed SN. Regardless, he estimated that the rate of such transients was favorable for detection by future surveys. His rate estimate turned out to be correct, but he unfortunately passed away just 5 years before the detection of the first multiply imaged gravitationally lensed SN.

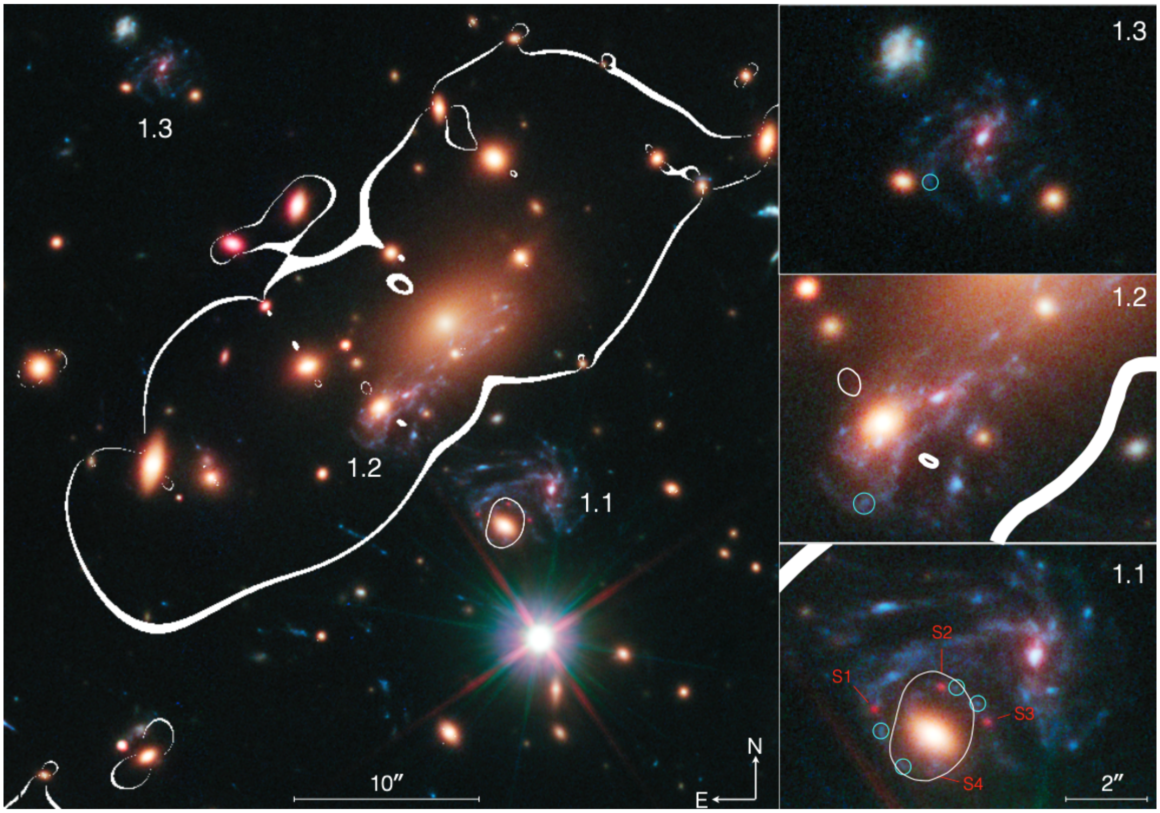


Figure 2. Composite image of SN Refsdal and massive galaxy cluster MACS J1149.6+2223 that acts as a lens. The three images of the host galaxy at z = 1.49 are labeled 1.1, 1.2, and 1.3. SN Refsdal was first detected in 1.1, where a member galaxy of the lensing cluster further splits the SN into 4 sub-images. Critical curves around the lens are drawn in white. Adapted from (Kelly et al. 2015).

4.1. *SN Refsdal*

On 10 November 2014, Patrick Kelly and collaborators pointed the *Hubble Space Telescope* (HST) at the galaxy cluster MACS J1149.6+2223 with the goal of studying the distant galaxies that are magnified by the lens this cluster creates. Fortuitously, they detected a quadruply imaged point source forming an "Einstein Cross" around a z = 0.54 elliptical galaxy within the cluster (Kelly et al. 2015). By referencing archival HST images of the cluster, they confirmed that this source was not present in the past. Therefore, this was the first observed instance of a multiply imaged gravitationally lensed transient (Kelly et al. 2015). They named it "SN Refsdal" in honor of the Norwegian astrophysicist who first predicted the existence of such an astonishing natural spectacle.

The complex lensing system in which SN Refsdal was discovered is shown in Figure 2. The composite image is striking, as the z = 1.49 host galaxy of SN Refsdal is duplicated three times around the cluster. However, in November 2014, SN Refsdal was detected in only one of these images. By sheer cosmic coincidence, a member galaxy of the lensing cluster sits in front of this image of the spiral arm in which

SN Refsdal occurred. The result is that SN Refsdal is further duplicated into 4 sub images around this elliptical galaxy forming an Einstein Cross (Kelly et al. 2015).

Analysis of the light curve of SN Refsdal revealed it to be inconsistent with that of a SN Ia (Kelly et al. 2015). This negates its usefulness to SN Ia cosmology, as it is not a standard candle. However, SN Refsdal could still be used for time delay cosmography if time delays could be constrained. Kelly's team modeled the lens system and predicted that SN Refsdal had appeared years ago in image 1.3, but went undetected (Kelly et al. 2015). The four detected sub-images of SN Refsdal around the elliptical lensing cluster member were predicted to have time delays on the order of days. Due to the non-standard and slow evolving nature of the SN, the time delay between these images could not be constrained well enough to measure H_0 (Kelly et al. 2015). All was not lost however, as models predicted that the image of the host with the longest null geodesics was image 1.2. This means that at the time of the detection of SN Refsdal in image 1.1, the photons that would form SN Refsdal in image 1.2 were still on their way to Earth. Kelly's models predicted that SN Refsdal would reappear in image 1.2 within a year to a decade (Kelly et al. 2015). By measuring the time delay between the arrival of SN Refsdal in images 1.1 and 1.2, time delay cosmography could be attempted for the first time in history.

With bated breath, astronomers awaited the reemergence of SN Refsdal. Just over a year later on 11 December 2015, SN Refsdal appeared in image 1.2 just as expected (Kelly et al. 2023). Without the lensing cluster member in front of its spiral arm as in image 1.1, SN Refsdal appeared in this image as a single point source rather than 4. Considering 8 models of the lensing cluster, they found $H_0 = 64.8^{+4.4}_{-4.3}\,\mathrm{km\,s^{-1}Mpc^{-1}}$ (Kelly et al. 2023).

4.2. *SN Requiem*

While analyzing archival HST images of the strong lensing cluster MACS J0138.0-2155 from 2019 and 2016, Steven Rodney and collaborators noticed that three point sources visible in the 2016 data were no longer present in the 2019 data (Rodney et al. 2021). Quite by accident, they had discovered a gravitationally lensed supernova at z = 1.95 whose images had long since faded. Because of this, follow up measurements could not be performed and constraints on the explosions properties were poor. Luckily, after modeling the lensing cluster, Rodney's team estimated that a fourth image of this SN would re appear in roughly the year 2037 (Rodney et al. 2021). This is roughly the year at which HST will be de-orbited and burn up in the atmosphere. In honor of the space telescope that made this discovery possible, they named the SN "Requiem" (Rodney et al. 2021).

SN Requiem was classified as a Type Ia SN using photometry derived from the three images of the SN. In principle, observations of a strongly lensed SN of this type are vital for constraining H_0. Unfortunately, SN Requiem was discovered in archival images long after it had faded. Consequently there could be no follow up observations to constrain the SN's properties and time delays further. The short predicted time delays between the three detected images of the SN yield large error bars on H_0 given the single epoch nature of the observation. However, astronomers need not despair at this wasted potential, for the best of SN Requiem is yet to come!

Upon the reemergence of the SN, H_0 can be constrained with unprecedented precision. The extremely long time delay ($\sim$ 20 years) of Requiem's fourth image allows for extraordinarily precise measurements of H_0, despite its poorly constrained explosion parameters (Rodney et al. 2021). The HST images from which Requiem was discovered are found in Figure 3. Here it can be seen that the red host of Requiem is warped into a partial Einstein Ring below and to the right of the cluster. This ring is actually composed of 3 separate images of the host, each containing Requiem in the 2016 images which is absent from the 2019 images. A fourth image of the host is seen above and to the left of the cluster, where Requiem is not seen in either the 2016 or 2019 images. This is where Requiem is predicted to reappear in 2037 (Rodney et al. 2021). A fifth and final image of the host is seen at the center of the cluster, where the host is strongly de-magnified. Requiem is expected to appear at this position after it appears in image 4. However, as a result of the de-magnification of this image, the SN is expected to go undetected here (Rodney et al. 2021).

4.3. *SN H0pe*

One of the "Guaranteed Time Observations" executed during Cycle 1 program of the *James Webb Space Telescope* (JWST) was the Prime Extragalactic Areas for Reionization and Lensing Science (PEARLS) project. JWST is the perfect instrument for this project, as its resolution in the near to mid infrared is unparalleled. As part of this project, JWST observed the strong lensing double-cluster PLCK G165.7+67.0 at z = 0.35 (Frye et al. 2024). In the NIRCam images from this observation, Brenda Frye, Massimo Pascale, and collaborators discovered three point sources that were absent from archival images of this cluster. This NIRCam image is shown in Figure 4. Measurements of spectral lines from the transient's host galaxy reveal its redshift to be z = 1.79. By modeling the gravitational lens using the LTM method, the time delay between these three point sources can be predicted (Frye et al. 2024). This model of the cluster did not predict that the transient would appear at a future time in a different image, however this particular transient still had remarkable potential to constrain H_0. With the predicted time delay in hand, each of these point sources are functionally three observations of the same SN at different phases (i.e. three points on the light curve of this SN). By comparing this light curve to other SNe, it was determined with 90% certainty that this transient was a Type Ia SN (Frye et al. 2024). SNe Ia are very well behaved transients, by which I mean their light curves evolve predictably. Furthermore, they are standard candles which reveal the luminosity distance of their host and allow the mass-sheet degeneracy of the lens to be broken. These factors made this SN an exceptional object for constraining H_0,

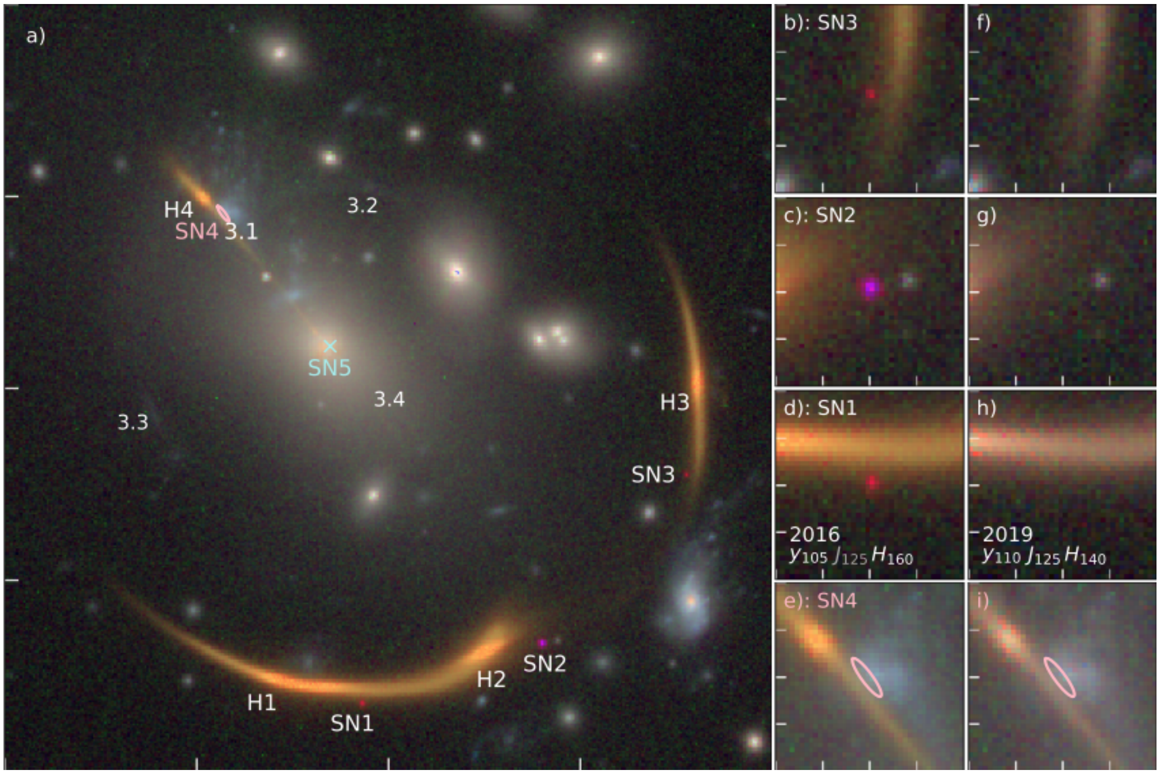


Figure 3. Shown here is the HST composite image of SN Requiem and the lensing cluster MACS J0138.0-2155. The images of Requiem are labeled SN1-5 and the images of the host are labeled H1-4. The first column to the right of (a) shows close up crops of the SN location in 2016, while the rightmost column shows those same regions in 2019. Requiem is not seen at the SN4 location in 2016 or 2019, but the most likely site of its reemergence in 2037 is marked by an ellipse. The labels 3.1-3 mark [O II] emitters used to constrain the lensing potential. Adapted from Rodney et al. (2021).

and so Frye and collaborators named this transient "SN H0pe", with the optimistic wish that it may mitigate the Hubble tension.

In a flurry of Directors Discretionary Time (DDT) proposals, further observations of SN H0pe using JWST were proposed for, approved, and promptly undertaken. Armed with high resolution spectra and precision photometry, Pascale and Frye set out to measure H_0 with the mass-sheet degeneracy of the lens broken for the first time (Pascale et al. 2025). The spectroscopic and photometric follow up confirmed that SN H0pe was a "normal" SN Ia, constraining its inherent luminosity to a small range of values derived from the well modeled Philips Relation (Phillips 1993). Using the inferred absolute magnitude of SN H0pe to constrain their lensing models, Pascale, Frye, and collaborators arrived at a value of $H_0 = 75.7^{+8.1}_{-5.5}$ $\mathrm{km\,s^{-1}}$ $\mathrm{Mpc^{-1}}$ (Pascale et al. 2025).

4.4. *SN Encore*

The host of SN Requiem is remarkable because it is the brightest known galaxy at $z \simeq 2$ on account of the gravitational lens magnifying it. It is therefore a fantastic target for JWST observation. During the first cycle of JWST observations, Andrew

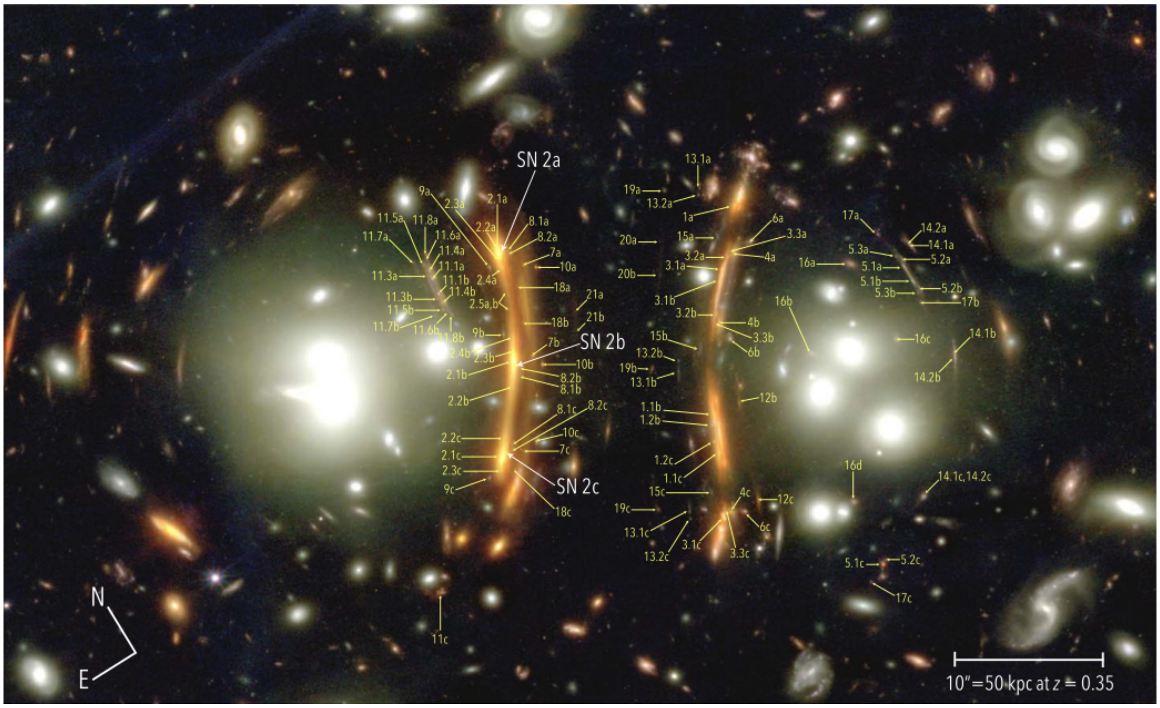


Figure 4. Shown here is the JWST NIRCam image of the lensing double-cluster PLCK G165.7+67.0 as well as the triply imaged SN H0pe and its host galaxy. H0pe's images are seen in the lensed arc just to the right of the leftmost cluster where they are labeled SN 2a-c. The other arc, to the right of SN H0pe's triply imaged host, is actually separate galaxy at a higher redshift than SN H0pe. Adapted from Frye et al. (2024).

Newman proposed to observe this galaxy in order to characterize early galactic evolution. Newman's proposal made no mention of SN Requiem or lensed transient studies (Newman et al. 2021). The proposal was accepted, and the observations were taken on 17 November, 2023. Astonishingly, three point sources were identified in this NIRCam image that were not visible in archival HST images. No, this was not SN Requiem prematurely reemerging. Rather, this was an entirely different SN in the same host galaxy as SN Requiem (Pierel et al. 2024)! Appropriately, they named this transient "SN Encore". Justin Pierel along with Andrew Newman and collaborators recognized the extraordinary scientific potential of SN Encore and promptly submitted a DDT proposal for three more JWST observations. The lensed transient collaboration "Lenswatch" triggered HST observations of the SN to complement the JWST data. These follow up observations confirmed SN Encore to be of Type Ia, just like SN Requiem. It is hard to overstate how fortuitous it is for two SNe Ia to occur in the same strongly lensed host. Pierel and collaborators estimate the chance of such an occurrence to be ∼3% (Pierel et al. 2024). This event is certainly lucky, but the odds are not astronomical. What makes this occurrence truly serendipitous is the scientific value that two SNe Ia from the same host provides. The standard candle nature of SNe Ia provides a means of cross checking and verifying the luminosity distance of the host. Furthermore, SN Encore provides another set of time delay measurements at different positions in the lensed host, allowing for even stronger constraints to be

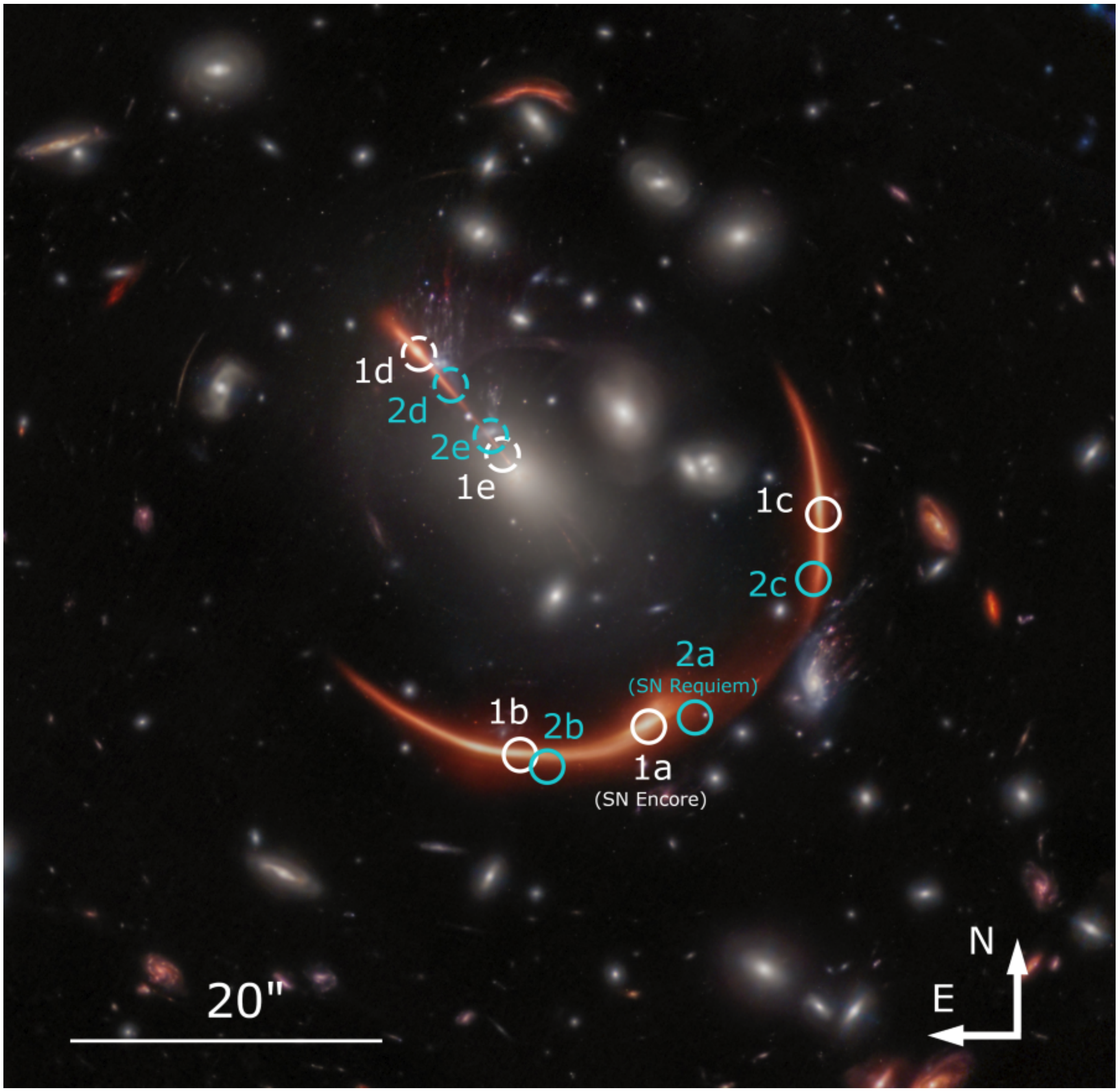


Figure 5. Shown here is a combined JWST and HST image of the lensing cluster MACS J0138.0-2155 and SN Encore. The labels 1a-c show the locations where SN Encore was detected. The labels 2a-c show the locations where SN Requiem were detected in archival HST imagery, though Requiem is not visible in this image. The locations where SNe Encore and Requeim are expected to re-appear in the 2030s are labeled 1d-e and 2d-e respectively. Adapted from Ertl et al. (2025).

placed on the mass distribution in the lens (Ertl et al. 2025). Through more accurate modeling of the lensing cluster and SN host system, time delay cosmography from the future reemergence of SN Requiem and SN Encore can be conducted with truly exceptional accuracy. Studies that derive H_0 and its uncertainties from the existing measurements of SN Encore are still underway and will likely be published in the newx few years, but the true spectacle will have to wait until the 2030s when both SN Requiem and SN Encore return to dazzle astronomers.

5. THE PRESENT AND FUTURE OF TIME DELAY COSMOGRAPHY

5.1. *Present*

To date, only two multiply imaged gravitationally lensed supernovae have been used to measure H_0 through the technique of time delay cosmography: SNe Refsdal and H0pe. Despite high hopes for this technique, these measurements have not solved the crisis in cosmology. In Figure 6, these measurements are compared to other measurements of H_0 using other techniques. Broadly, the H_0 values derived from time delay cosmography only add to the Hubble Tension. The H_0 values from SN H0pe have a 1.3-1.5σ tension with the values from SN Refsdal (Pascale et al. 2025). Furthermore, the values from SN H0pe have a 1.4-1.5σ tension with the H_0 measurements from analysis of the Cosmic Microwave Background (CMB) and Big Bang Nucleosynthesis (BBN) (Pascale et al. 2025). However, SN H0pe's values for H_0 are at least in general agreement with those derived from SNe Ia in the local universe. Overall, very little can be said about the Hubble Tension given the present state of time delay cosmography.

5.2. *Future*

5.2.1. *Re-emergence of SNe Requiem an Encore*

It is plain to see that time delay cosmography is still in its infancy; the error bars for this method are the largest on the plot in Figure 6. By refining lens models and taking more measurements these error bars can be refined, however some of the error is inherent to the lens. Multiply imaged SNe with short time delays between images yield higher errors, as the error bars on the time delays between images are a significant fraction of the time delays themselves. This can be seen in Figure 6, where the error bars on H_0 from SN Refsdal are smaller than those for SN H0pe. SN Refsdal had a time delay of $\sim$1 year, while SN H0pe's images were delayed by $\sim$50-100 days. This is why SNe Requiem and Encore are such promising objects for time delay cosmography: their time delays are on the order of $\sim$20 years. By constantly monitoring their host galaxy for their re-emergences, tight constraints can be placed on their time delay, within 1% of their 20 year baseline (Rodney et al. 2021).

5.2.2. *The Era of Deep All Sky Surveys*

In the modern era, most supernovae are discovered by all sky surveys such as ASAS-SN, ZTF, and Pan-STARRS. In contrast to this, none of the gravitationally lensed SNe discussed in this paper were discovered by this method. Instead, SNe Refsdal, Requiem, H0pe, and Encore were all discovered by chance in deep images from space telescopes. While they have been magnified greatly by gravitational lenses, these SNe are still too dim to be discovered by currently operational all sky surveys. This paradigm is set to change in the coming years when the Large Synoptic Survey Telescope (LSST) comes online.

For time delay cosmography to yield H_0 measurements with useful constraints, the time delay between images must be precisely measured. In practice, this requires the SN to be detected early so that follow up observations can be triggered. Thus, the best surveys to detect these transients must be deep, high cadence, and wide field.

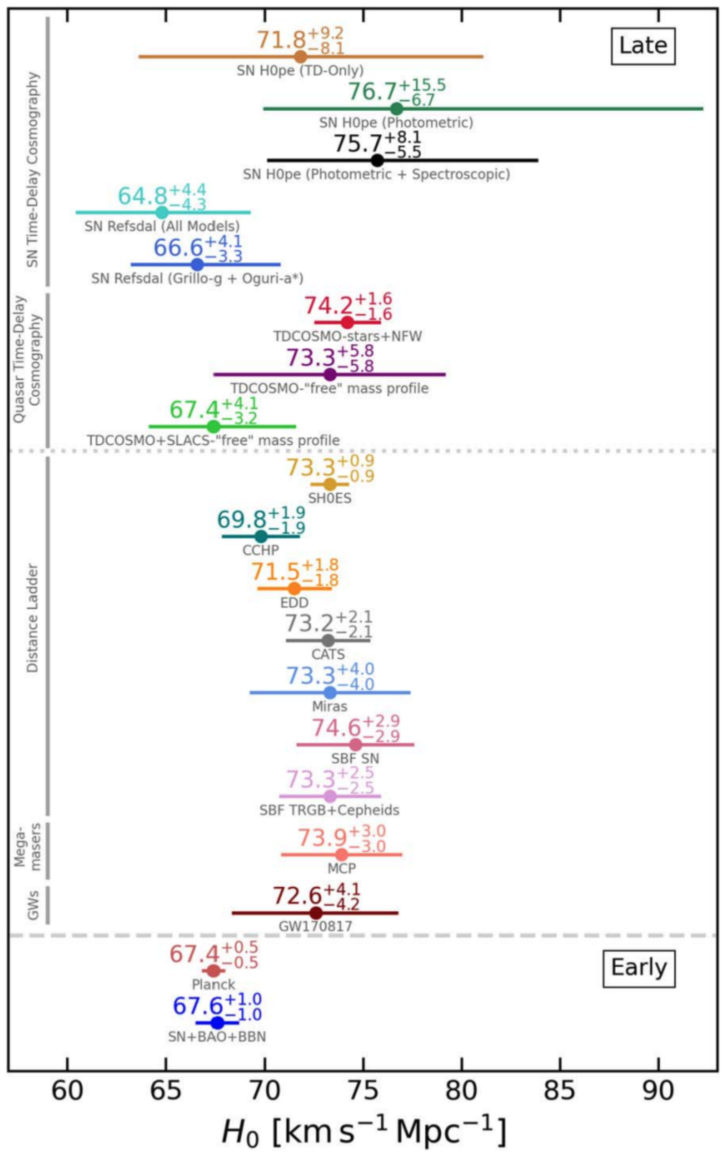


Figure 6. Shown here are all measurements of H_0 derived from time delay cosmography of multiply imaged lensed SNe alongside various measurements of H_0 from other experiments. These are roughly divided into "Early" measurements, such as those derived from the CMB, and "Late" measurements such as those derived from low redshift SNe Ia. Adapted from (Pascale et al. 2025).

That is, they must image large portions of the sky with small delays between images while being sensitive to very small changes in brightness. LSST is set to do just this. ZTF is a modern wide field survey that has a limiting magnitude of $\sim$20.5 (Bellm & Kulkarni 2017), while LSST will have a limiting magnitude of $\sim$ 23.5 (Oguri & Marshall 2010). This means that the dimmest transient LSST can detect would need to be $\sim$13$\times$ as bright for ZTF to detect it. As a result, LSST is expected to detect $\sim$ 100 gravitationally lensed SNe over its 10 year lifetime (Oguri & Marshall 2010). By detecting gravitationally lensed SNe early, tighter constraints can be placed on the epoch of emergence for the images, greatly reducing the errors on H_0 measurements using time delay cosmography.

6. CONCLUSION

Though the theory of time delay cosmography dates back over 60 years, measuring H_0 with the warped and delayed images of gravitationally lensed SNe has only become practical in the last decade. The potential of this method to provide answers to cosmological questions is enormous, as it provides an independent angle of inquiry from the typical tools of the CMB and standard candles. The preliminary results of time delay cosmography are not exactly reassuring, but this does not mean the method should be abandoned. Rather it implies we must bide our time. Time is required for deeper surveys like LSST to begin operation, and time is required for the photons from SNe Requiem and Encore to follow a longer path through the cosmic telescope and to Earth, where they will deliver the cosmological insight we seek. In the meantime, time delay cosmography can be continually refined by enhancing models of known gravitational lenses. Flagship space telescopes like JWST will continue to provide vital information on the mass distribution in these lenses through high resolution imagery and spectroscopy. Perhaps in one of these new images, astronomers will be delighted to find multiple point sources that are absent from the archival imagery.

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